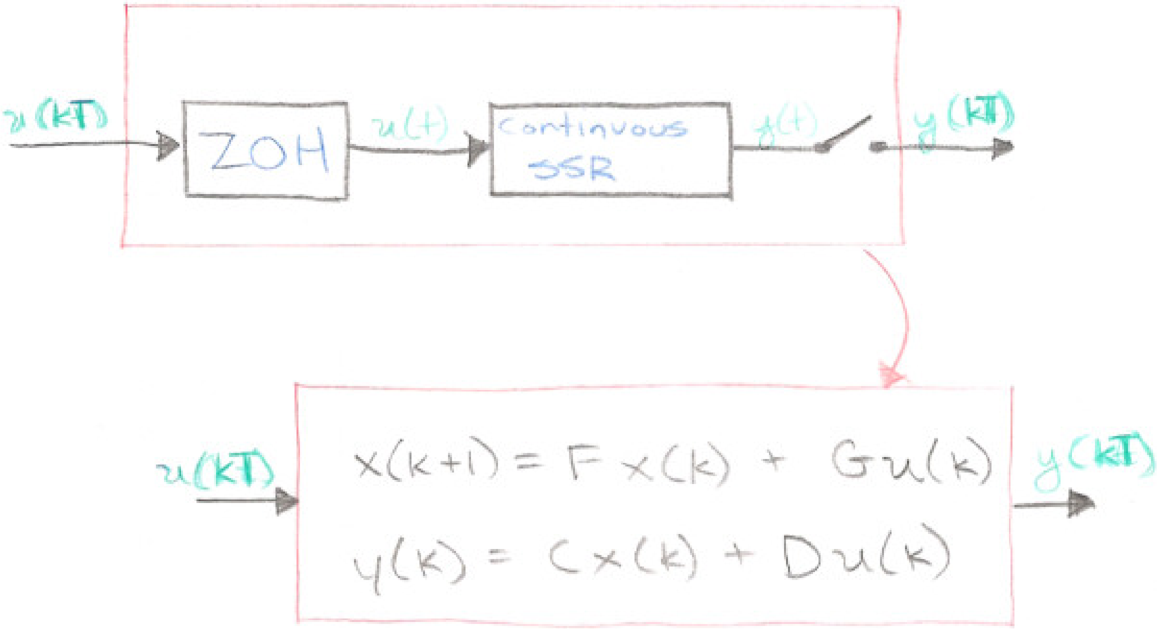
DISCRETE TIME SYSTEMS



$$F = e^{AT}$$

$$G = \int_0^T e^{A\tau} B \, d\tau = A^{-1}(e^{AT} - I)B$$

the eigen values of e^{AT} are $e^{\lambda_i T}$

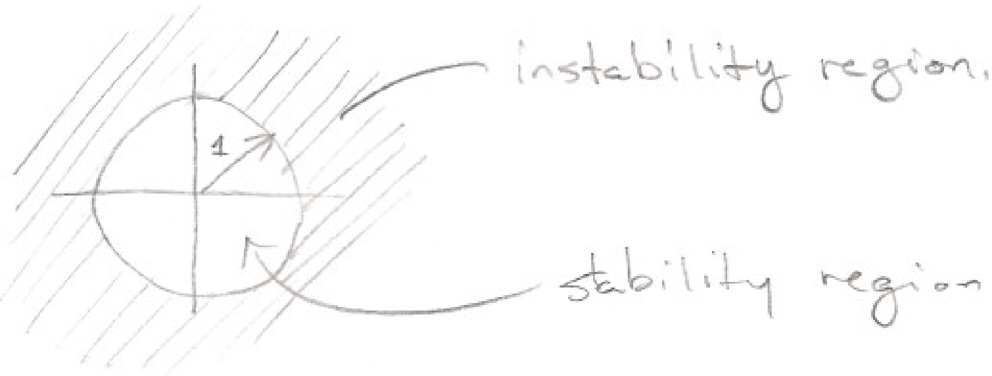
Ex: let $\lambda_i = \sigma + j\omega$. where λ_i is an eigen value of A.

$$e^{\lambda_i T} = e^{\sigma T}(\cos\omega T + j\sin\omega T)$$

$$|e^{\lambda_i T}| = |e^{\sigma T}| \underbrace{|\cos\omega T + j\sin\omega T|}_{=1} = e^{\sigma T}$$

if $\alpha > 0$ then $|e^{\alpha_i T}| > 0$

if $\alpha < 0$ then $|e^{\alpha_i T}| < 0$



To design controller

$$u(k) = -K x(k)$$

<u>note</u>: very similiar to continuous design.

CONTROLABILITY

$$x(k+1) = F x(k) + G u(k) \qquad x(k) \in \mathbb{R}^n$$

$$\text{rank}(\mathcal{C}) = n$$

$$\mathcal{C} = \begin{bmatrix} G & FG & \cdots & F^{n-1} G \end{bmatrix}$$

OBSERVABILITY

$$\text{rank}(O) = n$$

$$O = \begin{bmatrix} C \\ CF \\ C'F^{n-1} \end{bmatrix}$$

CONTROL DESIGN

same except what we want $\bar{A}$ to be

Ex:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$T = 0.1\,s$$

design discrete time state feedback

$$u(k) = -Kx(k)$$

such that the equivelent CT system will have a damping ratio of 0.5 and natural freq of 1 radian per second.

$$F = e^{At} = \begin{bmatrix} 1.0047 & 0.09078 \\ 0.09078 & 0.8231 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.00469 \\ 0.09078 \end{bmatrix}$$

with $u(k) = -Kx(k)$ ...

$$x(k+1) = (F - GK)\,x(k)$$

from $\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$

we find $\lambda_{1,2} = -0.5 \pm j0.866$

∴ the desired eigen values for the discrete time system (a) are

$$\bar{\lambda}_1 = e^{\lambda_1 T}$$
$$\bar{\lambda}_2 = e^{\lambda_2 T}$$

$$\bar{\lambda}_{1,2} = e^{-0.5T}\left(\cos(0.866T) + j\sin(0.866T)\right)$$
$$= 0.95 \pm j0.082$$

these are the roots of the desired system.